CR11 - Graded exercise sheet - Week 1

A *binary string* is a finite sequence $b_0 b_1 \dots b_{k-1}$ of bits $b_i \in \{0,1\}$. The *concatenation* of two strings σ and τ is denoted by $\sigma\tau$. One can code binary strings into integers by multiple ways. For example, the string $b_0 b_1 \dots b_{k-1}$ can be coded by $p_0^{b_0} p_1^{b_1} \dots p_{k-1}^{b_{k-1}}$, where p_i is the ith prime number. The actual representation of strings into integers does not matter, as long as the coding is computable. One can then define a set of strings to be computable if the corresponding set of integers is computable.

Exercise 1.1: /4

Given two sets of strings A and B, the *concatenation* of A and B is the set $AB = \{\sigma\tau : \sigma \in A \text{ and } \tau \in B\}$. Are the co-c.e. sets of strings closed under concatenation?

A partial computable function Φ_e is *strictly partial* if it is not total. In other words, it is strictly partial if $\Phi_e(x) \uparrow$ for some $x \in \mathbb{N}$.

Exercise 1.2: /6

Show that the set $\mathsf{Part} = \{e : \Phi_e \text{ is strictly partial }\}$ is $\emptyset'$-c.e.
Hint: consider the set $A = \{(e,x) : \forall y \Phi_e(x)[y] \uparrow\}$.

Recall that a set A is $\emptyset'$-c.e. if it is the domain of a partial $\emptyset'$-computable function.

Exercise 1.3: /10

Show that a set $A \subseteq \mathbb{N}$ is $\emptyset'$-c.e. iff there is a total computable function $f : \mathbb{N} \times \mathbb{N} \to \{0,1\}$ such that for every x, $x \in A$ iff $\lim_y f(x,y) = 1$. Note that if $x \notin A$, then the limit $\lim_y f(x,y)$ does not necessarily exist.