CR11 - Graded exercise sheet - Week 3

Exercise 3.1: /10

Build a *c.e.* infinite tree $T \subseteq 2^{<\mathbb{N}}$ such that every path computes $\emptyset'$.

A path P in a tree $T \subseteq 2^{<\mathbb{N}}$ is *isolated* if there is some initial segment $\sigma \prec P$ such that $[\sigma] \cap [T] = \{P\}$. In other words, P is isolated if there is some initial segment $\sigma \prec P$ such that for every $\tau \prec P$ with $|\sigma| \leq |\tau|$, exactly one of $\tau 0$ and $\tau 1$ has an infinite subtree below it. A path which is not isolated is a *limit point*.

Exercise 3.2: /10

Let $T \subseteq 2^{<\mathbb{N}}$ be a computable tree such that $[T]$ has exactly one limit point P. Show that $P \leq_T \emptyset''$. Hint: try to define a $\emptyset''$-computable infinite subtree $S \subseteq T$ which removed all the isolated paths of T, so that P becomes an isolated path of S.