

CR11 - Graded exercise sheet - Week 4

A function $f : \mathbb{N} \rightarrow \{0, 1\}$ is DNC_2 if for every $e \in \mathbb{N}$, $f(e) \neq \Phi_e(e)$. A function $f : \mathbb{N} \rightarrow \{0, 1\}$ is $\text{DNC}_2(X)$ if for every $e \in \mathbb{N}$, $f(e) \neq \Phi_e^X(e)$.

Exercise 4.1: /5

Show that if f is $\text{DNC}_2(X)$, then $f \geq_T X$.

Exercise 4.2: /8

Show that if X computes a DNC_2 function, then X computes a DNC_2 function f such that $f \equiv_T X$.

A Turing degree \mathbf{d} is *minimal* if the only degree strictly below it is $\mathbf{0}$, the degree of computable sets.

Exercise 4.3: /7

Show that $\{X \oplus Y : X \text{ is } \text{DNC}_2 \text{ and } Y \text{ is } \text{DNC}_2(X)\}$ is a Π_1^0 class. Deduce that there is no minimal PA degree.