

Bibliography

- [1] Barry S. Cooper. *Computability Theory*. CRC Press, 2003 (cited on page 3).
- [2] Robert I. Soare. ‘Turing Computability’. In: *Theory and Applications of Computability*. Springer (2016) (cited on pages 3, 33, 34).
- [3] B Monin and L Patey. ‘Calculabilité: Degrés Turing, Théorie Algorithmique de l’aléatoire, Mathématiques à Rebours’. In: *Hypercalculabilité, Calvage et Mounet* (2022) (cited on page 3).
- [4] Stephen G. Simpson. *Subsystems of Second Order Arithmetic*. Vol. 1. Cambridge University Press, 2009 (cited on pages 3, 71, 76, 103).
- [5] Damir D Dzhafarov and Carl Mummert. *Reverse mathematics: problems, reductions, and proofs*. Springer Nature, 2022 (cited on page 3).
- [6] Denis R Hirschfeldt. ‘Slicing the Truth’. In: *Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore* 28 (2015). Publisher: World Scientific Publishing (cited on pages 3, 81).
- [7] Joseph R. Shoenfield. ‘On Degrees of Unsolvability’. In: *Annals of Mathematics* (1959). Publisher: JSTOR, pp. 644–653 (cited on pages 15, 78, 90).
- [8] Carl G. Jockusch Jr. and Robert I. Soare. ‘ Π_1^0 Classes and Degrees of Theories’. In: *Trans. Amer. Math. Soc.* 173 (1972), pp. 33–56. doi: [10.2307/1996261](https://doi.org/10.2307/1996261) (cited on pages 17, 38, 88).
- [9] David Seetapun and Theodore Slaman. ‘On the Strength of Ramsey’s Theorem’. In: *Notre Dame Journal of Formal Logic* 36.4 (1995). Publisher: University of Notre Dame, pp. 570–582 (cited on pages 20, 23, 24).
- [10] Damir D Dzhafarov and Carl G. Jockusch. ‘Ramsey’s Theorem and Cone Avoidance’. In: *The Journal of Symbolic Logic* 74.2 (2009). Publisher: Cambridge University Press, pp. 557–578 (cited on pages 20, 21).
- [11] Jiayi Liu. ‘ RT_2^2 does not Imply WKL_0 ’. In: *The Journal of Symbolic Logic* (2012). Publisher: JSTOR, pp. 609–620 (cited on pages 20, 50–52, 54, 56).
- [12] Carl G. Jockusch and Frank Stephan. ‘A Cohesive Set which is not High’. In: *Mathematical Logic Quarterly* 39.1 (1993). Publisher: Wiley Online Library, pp. 515–530 (cited on pages 21, 42, 103).
- [13] Wei Wang. ‘Some Logically Weak Ramseyan Theorems’. In: *Advances in Mathematics* 261 (2014), pp. 1–25 (cited on pages 24, 104).
- [14] Carl G. Jockusch. ‘Ramsey’s Theorem and Recursion Theory’. In: *The Journal of Symbolic Logic* 37.2 (1972). Publisher: Cambridge University Press, pp. 268–280 (cited on pages 24, 32, 103).
- [15] Denis R. Hirschfeldt and Carl G. Jockusch. ‘On Notions of Computability-Theoretic Reduction between Π_2^1 Principles’. In: *J. Math. Log.* 16.1 (2016), pp. 1650002, 59. doi: [10.1142/S0219061316500021](https://doi.org/10.1142/S0219061316500021) (cited on page 25).
- [16] Rod Downey et al. *Relationships between Computability-Theoretic Properties of Problems*. 2019 (cited on pages 26, 28, 29).
- [17] Robert M. Solovay. ‘Hyperarithmetically encodable sets’. In: *Trans. Amer. Math. Soc.* 239 (1978), pp. 99–122. doi: [10.2307/1997849](https://doi.org/10.2307/1997849) (cited on page 27).
- [18] Webb Miller and D. A. Martin. ‘The degrees of hyperimmune sets’. In: *Z. Math. Logik Grundlagen Math.* 14 (1968), pp. 159–166. doi: [10.1002/malq.19680140704](https://doi.org/10.1002/malq.19680140704) (cited on page 27).
- [19] E. Herrmann. ‘Infinite chains and antichains in computable partial orderings’. In: *J. Symbolic Logic* 66.2 (2001), pp. 923–934. doi: [10.2307/2695053](https://doi.org/10.2307/2695053) (cited on page 32).
- [20] R. G. Downey. ‘Computability theory and linear orderings’. In: *Handbook of recursive mathematics*, Vol. 2. Vol. 139. Stud. Logic Found. Math. North-Holland, Amsterdam, 1998, pp. 823–976. doi: [10.1016/S0049-237X\(98\)80047-5](https://doi.org/10.1016/S0049-237X(98)80047-5) (cited on page 32).
- [21] Denis R. Hirschfeldt and Richard A. Shore. ‘Combinatorial Principles Weaker Than Ramsey’s Theorem for Pairs’. In: *Journal of Symbolic Logic* 72.1 (2007), pp. 171–206 (cited on pages 32, 40, 93).

- [22] Dana Scott. ‘Algebras of Sets Binumerable in Complete Extensions of Arithmetic’. In: *Proc. Sympos. Pure Math.* Vol. 5. 1962, pp. 117–121 (cited on page 35).
- [23] Clifford Spector. ‘On Degrees of Recursive Unsolvability’. In: *Ann. of Math. (2)* 64 (1956), pp. 581–592. doi: [10.2307/1969604](https://doi.org/10.2307/1969604) (cited on page 36).
- [24] Stephen C. Kleene and Emil L. Post. ‘The Upper Semi-Lattice of Degrees of Recursive Unsolvability’. In: *Annals of Mathematics* (1954). Publisher: JSTOR, pp. 379–407 (cited on page 37).
- [25] Peter A. Cholak, Carl G. Jockusch, and Theodore A. Slaman. ‘On the Strength of Ramsey’s Theorem for Pairs’. In: *The Journal of Symbolic Logic* 66.1 (2001). Publisher: Cambridge University Press, pp. 1–55 (cited on pages 42, 43, 46, 81).
- [26] Rod Downey et al. ‘A Δ_2^0 Set with no Infinite Low Subset in either it or its Complement’. In: *Journal of Symbolic Logic* 66.3 (2001), pp. 1371–1381 (cited on page 45).
- [27] C. T. Chong, Theodore A. Slaman, and Yue Yang. ‘The metamathematics of stable Ramsey’s theorem for pairs’. In: *J. Amer. Math. Soc.* 27.3 (2014), pp. 863–892. doi: [10.1090/S0894-0347-2014-00789-X](https://doi.org/10.1090/S0894-0347-2014-00789-X) (cited on pages 46, 96).
- [28] Stephen Flood. ‘Reverse mathematics and a Ramsey-type König’s lemma’. In: *J. Symbolic Logic* 77.4 (2012), pp. 1272–1280. doi: [10.2178/jsl.7704120](https://doi.org/10.2178/jsl.7704120) (cited on page 52).
- [29] Benoît Monin and Ludovic Patey. ‘Pigeons do not jump high’. In: *Advances in Mathematics* 352 (2019), pp. 1066–1095 (cited on pages 52, 114, 115, 119, 123).
- [30] Lu Liu. ‘Cone avoiding closed sets’. In: *Trans. Amer. Math. Soc.* 367.3 (2015), pp. 1609–1630. doi: [10.1090/S0002-9947-2014-06049-2](https://doi.org/10.1090/S0002-9947-2014-06049-2) (cited on pages 52, 60, 63).
- [31] Rodney G. Downey and Denis R. Hirschfeldt. *Algorithmic Randomness and Complexity*. Springer Science & Business Media, 2010 (cited on page 57).
- [32] André Nies. *Computability and Randomness*. Vol. 51. Oxford University Press, 2009 (cited on page 57).
- [33] Gregory J. Chaitin. ‘A Theory of Program Size Formally Identical to Information Theory’. In: *Journal of the ACM (JACM)* 22.3 (1975). Publisher: ACM New York, NY, USA, pp. 329–340 (cited on page 57).
- [34] Leonid A. Levin. ‘Laws of Information Conservation (Nongrowth) and Aspects of the Foundation of Probability Theory’. In: *Problemy Peredachi Informatsii* 10.3 (1974). Publisher: Russian Academy of Sciences, pp. 30–35 (cited on page 57).
- [35] Laurent Bienvenu, Ludovic Patey, and Paul Shafer. ‘On the Logical Strengths of Partial Solutions to Mathematical Problems’. In: *Trans. London Math. Soc.* 4.1 (2017), pp. 30–71. doi: [10.1112/tlm3.12001](https://doi.org/10.1112/tlm3.12001) (cited on page 64).
- [36] David B. Posner and Robert W. Robinson. ‘Degrees Joining to 0’. In: *Journal of Symbolic Logic* (1981). Publisher: JSTOR, pp. 714–722 (cited on page 66).
- [37] Carl G. Jockusch and Richard A. Shore. ‘Pseudo-Jump Operators. II: Transfinite Iterations, Hierarchies and Minimal Covers’. In: *The Journal of Symbolic Logic* 49.4 (1984). Publisher: JSTOR, pp. 1205–1236 (cited on page 66).
- [38] Stephen G. Simpson. ‘Partial Realizations of Hilbert’s Program’. In: *J. Symbolic Logic* 53.2 (1988), pp. 349–363. doi: [10.2307/2274508](https://doi.org/10.2307/2274508) (cited on page 71).
- [39] William W Tait. ‘Finitism’. In: *The Journal of Philosophy* (1981), pp. 524–546 (cited on page 71).
- [40] Theodore A Marcia J. Groszek Slaman. ‘On Turing Reducibility’. In: () (cited on pages 72, 87).
- [41] Petr Hájek and Pavel Pudlák. *Metamathematics of First-Order Arithmetic*. Perspectives in Mathematical Logic. Berlin: Springer-Verlag, 1998 (cited on pages 72–75, 84, 87, 97).
- [42] J. B. Paris and L. A. S. Kirby. Σ_n -Collection Schemas in Arithmetic. 1978. doi: [10.1016/s0049-237x\(08\)72003-2](https://doi.org/10.1016/s0049-237x(08)72003-2) (cited on pages 73, 75).
- [43] R. Kaye, J. Paris, and C. Dimitracopoulos. ‘On parameter free induction schemas’. In: *J. Symbolic Logic* 53.4 (1988), pp. 1082–1097. doi: [10.2307/2274606](https://doi.org/10.2307/2274606) (cited on page 73).
- [44] Charles Parsons. ‘On a Number Theoretic Choice Schema and its Relation to Induction’. In: *Intuitionism and Proof Theory (Proc. Conf., Buffalo, N.Y., 1968)*. North-Holland, Amsterdam, 1970, pp. 459–473 (cited on page 74).

- [45] Theodore A. Slaman. ‘ Σ_n -Bounding and Δ_n -Induction’. In: 132 (2004), pp. 2449–2449. doi: [10.1090/s0002-9939-04-07294-6](https://doi.org/10.1090/s0002-9939-04-07294-6) (cited on page 75).
- [46] Harvey Friedman. ‘Systems on Second Order Arithmetic with Restricted Induction I, II’. In: *Journal of Symbolic Logic* 41 (1976), pp. 557–559 (cited on page 76).
- [47] Harvey Martin Friedman. ‘Subsystems of Set Theory and Analysis’. PhD Thesis. Massachusetts Institute of Technology, 1967 (cited on page 76).
- [48] António M. Fernandes, Fernando Ferreira, and Gilda Ferreira. ‘Analysis in weak systems’. In: *Logic and computation*. Vol. 33. Tributes. Coll. Publ., [London], 2017, pp. 231–261 (cited on page 77).
- [49] Henry Towsner. ‘On maximum conservative extensions’. In: *Computability* 4.1 (2015), pp. 57–68 (cited on page 78).
- [50] Stephen G. Simpson and Rick L. Smith. ‘Factorization of polynomials and Σ_1^0 induction’. In: vol. 31. 2-3. Special issue: second Southeast Asian logic conference (Bangkok, 1984). 1986, pp. 289–306. doi: [10.1016/0168-0072\(86\)90074-6](https://doi.org/10.1016/0168-0072(86)90074-6) (cited on pages 81–83).
- [51] David R Belanger. ‘Conservation theorems for the cohesiveness principle’. In: *arXiv preprint arXiv:2212.13011* (2022) (cited on pages 82, 90, 92).
- [52] Jeffry L. Hirst. ‘Combinatorics in Subsystems of Second Order Arithmetic’. PhD thesis. Pennsylvania State University, Aug. 1987 (cited on page 82).
- [53] Marta Fiori-Carones et al. ‘An isomorphism theorem for models of Weak König’s Lemma without primitive recursion’. In: *arXiv preprint arXiv:2112.10876* (2021) (cited on pages 83, 86, 87).
- [54] Leszek Aleksander Kołodziejczyk and Keita Yokoyama. ‘Categorical characterizations of the natural numbers require primitive recursion’. In: *Ann. Pure Appl. Logic* 166.2 (2015), pp. 219–231. doi: [10.1016/j.apal.2014.10.003](https://doi.org/10.1016/j.apal.2014.10.003) (cited on page 84).
- [55] C. T. Chong and K. J. Mourad. ‘The degree of a Σ_n cut’. In: *Ann. Pure Appl. Logic* 48.3 (1990), pp. 227–235. doi: [10.1016/0168-0072\(90\)90021-5](https://doi.org/10.1016/0168-0072(90)90021-5) (cited on page 84).
- [56] Keita Yokoyama. ‘On conservativity for theories in second order arithmetic’. In: *Proceedings of the 10th Asian Logic Conference*. World Scientific. 2010, pp. 375–386 (cited on pages 87, 95).
- [57] C. T. Chong, Theodore A. Slaman, and Yue Yang. ‘ Π_1^1 -conservation of combinatorial principles weaker than Ramsey’s theorem for pairs’. In: *Adv. Math.* 230.3 (2012), pp. 1060–1077. doi: [10.1016/j.aim.2012.02.025](https://doi.org/10.1016/j.aim.2012.02.025) (cited on pages 87, 92, 93).
- [58] Richard Kaye. *Models of Peano Arithmetic*. 1991 (cited on page 87).
- [59] Petr Hájek. ‘Interpretability and fragments of arithmetic’. In: *Arithmetic, proof theory, and computational complexity (Prague, 1991)*. Vol. 23. Oxford Logic Guides. Oxford Univ. Press, New York, 1993, pp. 185–196 (cited on pages 88, 90).
- [60] Quentin Le Houérou, Ludovic Levy Patey, and Keita Yokoyama. ‘Conservation of Ramsey’s theorem for pairs and well-foundedness’. In: *arXiv preprint arXiv:2402.11616* (2024) (cited on pages 93, 98–100).
- [61] C. T. Chong, Steffen Lempp, and Yue Yang. ‘On the role of the collection principle for Σ_2^0 -formulas in second-order reverse mathematics’. In: *Proc. Amer. Math. Soc.* 138.3 (2010), pp. 1093–1100. doi: [10.1090/S0002-9939-09-10115-6](https://doi.org/10.1090/S0002-9939-09-10115-6) (cited on page 93).
- [62] Alexander P. Kreuzer and Keita Yokoyama. ‘On principles between Σ_1 - and Σ_2 -induction, and monotone enumerations’. In: *J. Math. Log.* 16.1 (2016), pp. 1650004, 21. doi: [10.1142/S0219061316500045](https://doi.org/10.1142/S0219061316500045) (cited on pages 96, 97).
- [63] Petr Hájek and Jeff Paris. ‘Combinatorial principles concerning approximations of functions’. In: *Arch. Math. Logik Grundlag.* 26.1-2 (1986/87), pp. 13–28. doi: [10.1007/BF02017489](https://doi.org/10.1007/BF02017489) (cited on page 96).
- [64] Vasco Brattka, Matthew Hendtlass, and Alexander P. Kreuzer. ‘On the uniform computational content of computability theory’. In: *Theory Comput. Syst.* 61.4 (2017), pp. 1376–1426. doi: [10.1007/s00224-017-9798-1](https://doi.org/10.1007/s00224-017-9798-1) (cited on page 103).
- [65] Benoît Monin and Ludovic Patey. *SRT22 does not imply COH in Omega-Models*. 2019 (cited on page 103).

- [66] Peter A. Cholak et al. ‘Free sets and reverse mathematics’. In: *Reverse mathematics 2001*. Vol. 21. Lect. Notes Log. Assoc. Symbol. Logic, La Jolla, CA, 2005, pp. 104–119 (cited on page 103).
- [67] Barbara F. Csima and Joseph R. Mileti. ‘The strength of the rainbow Ramsey theorem’. In: *J. Symbolic Logic* 74.4 (2009), pp. 1310–1324. doi: [10.2178/jsl/1254748693](https://doi.org/10.2178/jsl/1254748693) (cited on page 103).
- [68] Benoît Monin and Ludovic Patey. *The Weakness of the Pigeonhole Principle under Hyperarithmetical Reductions*. 2019 (cited on pages 104, 115, 116, 118).
- [69] Denis R. Hirschfeldt et al. ‘The Strength of some Combinatorial Principles Related to Ramsey’s Theorem for Pairs’. In: *Computational Prospects of Infinity, Part II: Presented Talks*, World Scientific Press, Singapore (2008), pp. 143–161. doi: [10.1142/9789812796554_0008](https://doi.org/10.1142/9789812796554_0008) (cited on page 107).
- [70] Wei Wang. ‘The Definability Strength of Combinatorial Principles’. In: *J. Symb. Log.* 81.4 (2016), pp. 1531–1554. doi: [10.1017/jsl.2016.10](https://doi.org/10.1017/jsl.2016.10) (cited on page 107).
- [71] Quentin Le Houérou, Ludovic Levy Patey, and Ahmed Mimouni. *The reverse mathematics of the pigeonhole hierarchy*. 2024 (cited on pages 109, 117, 118).
- [72] Donald A. Martin. ‘Classes of Recursively Enumerable Sets and Degrees of Unsolvability’. In: *Mathematical Logic Quarterly* 12.1 (1966). Publisher: Wiley Online Library, pp. 295–310 (cited on page 109).
- [73] Ludovic Patey. ‘Controlling Iterated Jumps of Solutions to Combinatorial Problems’. In: *Computability* 6.1 (2017), pp. 47–78. doi: [10.3233/COM-160056](https://doi.org/10.3233/COM-160056) (cited on page 112).
- [74] Benoit Monin and Ludovic Patey. ‘Partition genericity and pigeonhole basis theorems’. In: *J. Symb. Log.* 89.2 (2024), pp. 829–857. doi: [10.1017/jsl.2022.69](https://doi.org/10.1017/jsl.2022.69) (cited on pages 113, 115, 116).
- [75] Stephen Flood. ‘A packed Ramsey’s theorem and computability theory’. In: *Trans. Amer. Math. Soc.* 367.7 (2015), pp. 4957–4982. doi: [10.1090/S0002-9947-2015-06164-9](https://doi.org/10.1090/S0002-9947-2015-06164-9) (cited on page 113).