I am interested in the *constructive content* of mathematical reasoning. It is a well-established fact that some proofs involve more complex arguments than others. There are several ways to understand the notion of proof strength. Among them, *reverse mathematics* is a vast mathematical program whose goal is to study the logical strength of ordinary theorems in terms of set existence axioms. The choice of a computable base theory makes reverse mathematics a good framework to investigate both the proof-theoretic strength of theorems and their computational content.

I mainly work within the framework of reverse mathematics under a computational perspective. My research primarily focuses on the reverse mathematics of *combinatorial theorems*, and in particular on Ramsey's theorem and its consequences. My background is computability-theoretic, although I am also interested other areas of mathematical logic, such as proof theory, set theory, and model theory.

Background

I now briefly introduce the two frameworks in which I will state my main results.

Reverse mathematics

Reverse mathematics uses the language of second-order arithmetic, which happens to be sufficiently expressive to formalize in a natural way most of ordinary mathematics. The base theory is RCA_0, standing for Recursive Comprehension Axiom. RCA_0 contains the basic first-order Peano axioms, the Δ^0_1 comprehension scheme and the Σ^0_1 induction scheme. RCA_0 can be though as capturing *computable mathematics*.

Due to its goal of analyzing the logical strength of everyday life theorems, reverse mathematics is fundamentally an interdisciplinary research topic. The logical analysis of a theorem from a particular field requires to understand the deep mechanisms of the underlying theory. The base theory RCA_0 is composed of axioms, so that any proof over RCA_0 reveals the computable nature of theorems. Moreover, the search for optimal axioms sometimes leads to simpler proofs of the considered theorem. In that, all mathematics can potentially take benefit from reverse mathematics.

Far beyond its pragmatic applications to mathematics, reverse mathematics is of particular interest from a philosophical point of view. The early study of reverse mathematics revealed that most "ordinary", i.e., non set-theoretic, theorems are equivalent to one of five main subsystems, known as the Big Five [31]. These five basic systems correspond to well known philosophical approaches to mathematics. They are essentially similar to Bishop's constructivism; Hilbert's finitistic reductionism; the Predicativism of Weyl and Feferman; the Predicative Reductionism of Friedman and Simpson and Impredicativity as developed by Feferman and others. See Simpson [47, I.12] for a discussion.

We are in particular interested in models whose first-order part consists of the natural numbers. An *ω-structure* is a tuple $(\omega, \mathcal{S}, +, \cdot, 0, 1, <)$ where ω is the set of natural numbers, together with the standard arithmetic operations $+, \cdot, <$, and $\mathcal{S} \subseteq 2^\omega$ is a set of reals. An ω-structure is therefore fully specified by its second-order part $\mathcal{S}$. An ω-structure is a model of RCA_0 iff its second-order part is a *Turing ideal* $\mathcal{S}$, that is, $(\forall X, Y \in \mathcal{S})[X \oplus Y \in \mathcal{S}]$ and $(\forall Y \in \mathcal{S})(\forall X \leq_T Y)[X \in \mathcal{S}]$.

Computable reducibility

A proof of implication from a statement P to another statement Q is coarse, in that it does not take in account the number of applications of P in the proof of Q. There have been developments towards a refinement of the proof reducibility with more precise logics such as linear logic [20]. Recently, two main reducibility notions, namely, computable reducibility and uniform reducibility, appeared as a refinement of reverse mathematics and revealed subtle distinctions between theorems [5, 11, 23]. We shall introduce the former one.

Many theorems are Π^1_2 statements P of the form $(\forall X)[\Phi(X) \rightarrow (\exists Y)\Psi(X, Y)]$, where Φ and Ψ are arithmetic formulas. Such theorems can be thought of as *mathematical problems*. A P-*instance* is a set X such that $\Phi(X)$ holds. A *solution to* X is a set Y such that $\Psi(X, Y)$ holds. For example, König's lemma asserts that every infinite, finitely branching tree has an infinite path. An instance is an infinite, finitely branching tree T and a solution to T is an infinite path through T.

A Π^1_2 statement P is *computably reducible* to another Π^1_2 statement Q (written $\mathsf{P} \leq_c \mathsf{Q}$) if every P-instance X computes a Q-instance Y such that for every solution Z to Y, $Z \oplus X$ computes a solution to X. P is *uniformly reducible* to Q (written $\mathsf{P} \leq_u \mathsf{Q}$) if the reduction $\mathsf{P} \leq_c \mathsf{Q}$ is witnessed by two fixed Turing functionals.

When looking at ω-models, a reduction $\mathsf{Q} \leq_c \mathsf{P}$ can be seen as a proof that $\mathsf{RCA}_0 \vdash \mathsf{P} \rightarrow \mathsf{Q}$ where only one application of the P statement is allowed to compute a solution to a Q-instance. In this sense, computable reducibility is *finer* than provability over RCA_0.

Research accomplishments

In the past two decades, Ramsey theory emerged as one of the most important topics in reverse mathematics. Ramsey theory is a branch of mathematics studying the conditions under which some structure appears among a sufficiently large collection of objects. This theory provides a large class of theorems escaping the Big Five phenomenon, and whose strength is notoriously hard to gauge.

I investigated the strength of Ramsey's theorem and various consequences, such as the Erdős-Moser theorem, the free set and thin set theorems, the rainbow Ramsey theorem, the ascending descending sequence principle and the atomic model theorem, among others. The following sections cover materials appearing in my papers [2, 3, 19, 32, 33, 34, 42, 35, 36, 37, 38, 39, 40, 41, 43]. In what follows, WKL denotes the restriction of König's lemma to infinite binary trees, and WWKL denotes the restriction of WKL to trees T of positive measure, that is, such that $\lim_s \frac{|\{\sigma \in T : |\sigma| = s\}|}{2^s} > 0$. AMT denotes the atomic model theorem [24].

The colors in Ramsey's theorem

Ramsey theory plays an important role in reverse mathematics. Indeed, it provides many examples of theorems escaping the Big Five (see Montálban [31]). Among them, Ramsey's theorem (RT^n_k) asserts that every k-coloring of $[\mathbb{N}]^n$ admits an infinite homogeneous set.

A simple color-blindness argument shows that RT^n_k and RT^n_ℓ are equivalent over RCA_0 for each $n, k, \ell \geq 2$. However, whenever $k > \ell$, the proof that $\mathsf{RCA}_0 \vdash \mathsf{RT}^n_\ell \rightarrow \mathsf{RT}^n_k$ involves multiple applications of the statement RT^n_ℓ. Dorais, Dzhafarov, Hirst, Mileti and Shafer [11] tried to prove that $\mathsf{RT}^n_k \not\leq_u \mathsf{RT}^n_\ell$ whenever $k > \ell \geq 2$ and let it open as a "chief question". Rakotoniaina [45], Hirschfeldt and Jockusch [23] and I [43] answered this question independently. In fact, I proved the following stronger theorem, which also answers a question of Hirschfeldt [22].

Theorem 1 ([43]). *For every $n \geq 2$ and every $k > \ell \geq 2$, $\mathsf{SRT}^n_k \not\leq_c \mathsf{RT}^n_\ell$.*

Ramsey's theorem and the Erdős-Moser theorem

Ramsey's theorem for pairs admits two main decompositions.

First, RT^2_2 is equivalent to the conjunction of stable Ramsey's theorem for pairs (SRT^2_2) and cohesiveness (COH). The former is the restriction of RT^2_2 to colorings $f : [\mathbb{N}]^2 \to 2$ such that $\lim_s f(x,s)$ always exists. The latter asserts for every sequence of sets $R_0, R_1, \dots$ the existence of an infinite set C such that $C \subseteq^* R_i$ or $C \subseteq^* \overline{R}_i$ for every i.

Second, RT^2_2 is equivalent to the conjunction of the Erdős-Moser theorem (EM) and the ascending descending sequence principle (ADS). The former asserts that every infinite tournament admits an infinite transitive subtournament. The latter states the existence of an infinite ascending or descending sequence in every infinite linear order.

There has been a lot of recent literature around the weakness of the Erdős-Moser theorem. Lerman, Solomon and Towsner [26] proved that $\mathsf{RCA}_0 \wedge \mathsf{EM} \nvdash \mathsf{SADS}$ (where SADS is the restriction of ADS to linear orders of type $\omega + \omega^*$). I refined their proof to obtain $\mathsf{RCA}_0 \wedge \mathsf{EM} \nvdash \mathsf{STS}^2$ [33] (where STS^2 is the stable thin set theorem for pairs, defined below). Wang [49] enhanced this result to prove that $\mathsf{RCA}_0 \wedge \mathsf{EM} \wedge \mathsf{COH} \wedge \mathsf{WKL} \nvdash \mathsf{STS}^2 \vee \mathsf{SADS}$. Finally, I proved the following theorem, which strengthen all the above-mentioned results since AMT is a consequence of both SADS and STS^2 over RCA_0.

Theorem 2 ([32])**.** $\mathsf{RCA}_0 \wedge \mathsf{EM} \wedge \mathsf{COH} \wedge \mathsf{WKL} \nvdash \mathsf{AMT}$.

The free set and thin set hierarchies

Ramsey's theorem for n-tuples is equivalent to the arithmetic comprehension axiom whenever $n \geq 3$. Therefore, the hiearchy collapses at level 3. However, we can weaken Ramsey's theorem by allowing more colors in the solutions.

Given a coloring $f : [\mathbb{N}]^n \to k$, an infinite set is f-thin if it avoids at least one color. TS^n_k asserts the existence of an f-thin set for every k-coloring of $[\mathbb{N}]^n$, and TS^n is the same statement for functions $f : [\mathbb{N}]^n \to \mathbb{N}$. STS^2_k and STS^2 are the restrictions of TS^2_k and TS^2 to stable colorings, respectively.

The free set theorem (FS^n) is a strengthening of TS^n which asserts for every coloring $f : [\mathbb{N}]^n \to \mathbb{N}$ the existence of an f-free set. A set H is *f-free* if for every $\sigma \in [H]^n$, if $f(\sigma) \in H$ then $f(\sigma) \in \sigma$. In particular, if H is f-free, then for every $x \in H$, $H \setminus \{x\}$ is f-thin with witness x.

The free set and thin set theorems have been introduced by Friedman [18, 17] and studied by Cholak, Giusto, Hirst and Jockusch [7] and Wang [50] among others. Cholak et al. [7] and Montálban [31] asked whether TS^2 implies RT^2_2 over RCA_0. Cholak et al. [7] asked whether any of FS^2, $\mathsf{FS}^2 \wedge \mathsf{COH}$ and $\mathsf{FS}^2 \wedge \mathsf{WKL}$ implies RT^2_2 over RCA_0. Hirschfeldt [22] asked whether $\mathsf{FS}^2 \wedge \mathsf{WKL}$ implies any of SRT^2_2, ADS or CAC (the chain antichain principle) over RCA_0. I answered all these questions negatively through the following theorem.

Theorem 3 ([43])**.** *For every $k \geq 2$,* $\mathsf{RCA}_0 \wedge \mathsf{COH} \wedge \mathsf{WKL} \wedge \mathsf{EM} \wedge \mathsf{TS}^2_{k+1} \wedge \mathsf{FS} \nvdash \mathsf{STS}^2_k \vee \mathsf{SADS}$.

Many proofs of Ramsey's theorem for pairs involve weak König's lemma. The community naturally wondered whether WKL is really necessary to prove RT^2_2, and in particular whether RT^2_2 implies WKL over RCA_0. The question has been a long-standing open problem until Liu [27] proved that $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \nvdash \mathsf{WKL}$. He later refined is theorem by proving $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \nvdash \mathsf{WWKL}$ [28].

Hirschfeldt [22] asked whether any of FS^n, TS^n, FS or TS imply WKL over RCA_0 whenever $n \geq 3$. I answered these questions negatively with the following stronger theorem.

Theorem 4 ([35])**.** $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \wedge \mathsf{FS} \nvdash \mathsf{WWKL}$.

In fact, even the help of WWKL is not enough to obtain WKL.

Theorem 5 ([35])**.** $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \wedge \mathsf{FS} \wedge \mathsf{WWKL} \nvdash \mathsf{WKL}$.

Strengthening Ramsey's theorem for pairs

There were no natural theorem known to lie strictly between the arithmetic comprehension axiom and Ramsey's theorem for pairs until recently. There were however two good candidates.

The tree theorem (TT^n_k) asserts for every coloring of $[2^{<\mathbb{N}}]^n$ the existence of an infinite homogeneous subtree $T \subseteq 2^{<\mathbb{N}}$ isomorphic to the full binary tree $2^{<\mathbb{N}}$. Here, $[2^{<\mathbb{N}}]^n$ denotes the n-tuples of comparable nodes. The tree theorem was first analyzed by McNicholl [29] and by Chubb, Hirst, and McNicholl [9]. They proved that TT^2_2 lies between ACA and RT^2_2 over RCA_0, and left open whether any of the implications is strict. Further work was done by Corduan, Groszek, and Mileti [10] and Dzhafarov, Hirst and Lakins [12]. Montálban [31] asked whether RT^2_2 implies TT^2_2 over RCA_0. I answered negatively.

Theorem 6 ([41])**.** $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \wedge \mathsf{WKL} \nvdash \mathsf{TT}^2_2$.

Together with Dzhafarov, we closed the question by showing that TT^2_2 is strictly weaker than ACA in reverse mathematics.

Theorem 7 ([13])**.** $\mathsf{RCA}_0 \wedge \mathsf{TT}^2_2 \wedge \mathsf{WKL} \nvdash \mathsf{ACA}$.

Ramsey's theorem for pairs can be stated as $\omega \to (\omega)^2_2$, where $\alpha \to (\beta)^2_2$ is the statement "For every coloring $f\colon [L]^2 \to 2$, where L is a linear order of type α, there is a homogeneous set H such that $(H, \leq_L)$ has order type β". It turns out that ω and ω^* are the only countable order types α such that $\alpha \to (\alpha)^2_2$ holds. In particular, $\eta \to (\eta)^2_2$ does not hold, where η is the order type of the rationals. However, Erdős and Rado [14] proved that the partition relation $\eta \to (\aleph_0, \eta)^2$ holds. The statement $\eta \to (\aleph_0, \eta)^2$ asserts that for every coloring $f\colon [L]^2 \to 2$, where L is a linear order of order type η, there is either an infinite 0-homogeneous set or a 1-homogeneous set of order type η. Frittaion and I [19] studied the reverse mathematics of this Erdős-Rado theorem, which is arguably more natural than the tree theorem for pairs. The statement $\eta \to (\aleph_0, \eta)^2$ lies between ACA and RT^2_2. With Dzhafarov, we proved that $\eta \to (\aleph_0, \eta)^2$ is strictly stronger than Ramsey's theorem for pairs.

Theorem 8 ([13])**.** $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \wedge \mathsf{WKL} \nvdash \eta \to (\aleph_0, \eta)^2$.

Degrees bounding and universal instances

The theorems studied in reverse mathematics can be seen as collections of mathematical problems parameterized by their instances. The complexity of finding a solution to an instance depends on the instance. However, it happens that some theorems admit a *universal instance*, that is, a computable instance I such that for every computable instance J, every solution to I computes a solution to J. For example, the Π^0_1 class of completions of Peano arithmetic is a universal instance for WKL.

A common way to prove that a statement P admits no universal instance consists of using the notion of *degree bounding* P. A degree $\mathbf{d}$ *bounds* P if every computable P-instance has a $\mathbf{d}$-computable solution. Let $\mathcal{D}$ be a downward-closed class of Turing degrees. If every computable P-instance has a solution of degree in $\mathcal{D}$, but no degree in $\mathcal{D}$ bounds P, then P admits no universal instance.

Mileti [30] studied the degrees bounding Ramsey's theorem for pairs. He proved that no low_2 degree bounds SRT^2_2. I strengthened his result with essentially the same proof.

Theorem 9 ([37])**.** *Neither* SADS *nor* STS^2 *admit a* low_2 *bounding degree.*

However, much more surprisingly, the proof cannot be adapted to the Erdős-Moser theorem. In fact, the converse holds. I used the first jump and the second jump control techniques of Cholak, Jockusch and Slaman [8] to prove the following theorem.

Theorem 10 ([37])**.** *Every PA degree relative to* $\emptyset'$ *computes the jump of a degree bounding* EM*. In particular* EM *admits a* low_2 *bounding degree.*

Controlling iterated jumps

Many Ramsey-type theorems are proven using forcing constructions. Whether or not it is possible to design a notion of forcing whose forcing relation has the same complexity as the formulas it forces is a crucial question when dealing with Ramsey-type hierarchies. Indeed, the free set, thin set and the rainbow Ramsey theorems are known to satisfy Jockusch's bound [25], that is, the existence of a computable coloring over $[\mathbb{N}]^n$ with no Σ^0_n solution. Proving that every computable P^n instance has a low$_n$ solution would enable one to separate P^n from P^{n+1}, where P^n is any of FS^n, TS^n and RRT^n_2. In order to prove the existence of a low$_n$ solution, one has to decide Σ^0_n formulas in a $\emptyset^{(n)}$-effective construction.

All the forcing notions used to construct solutions to consequences of Ramsey's theorem for pairs are variants of Mathias forcing. However, Cholak, Dzhafarov, Hirst and Slaman [6] showed that the forcing relation for Mathias forcing does not admit the desired properties. In particular, the complexity of forcing a Π^0_2 formula is Π^0_3.

I designed new notions of forcing for cohesiveness and the Erdős-Moser theorem which admit a forcing relation with the good properties. Then, I used these notions to prove two conjectures of Wang [49].

Theorem 11 ([42]). COH *and* EM *admit preservation of the arithmetic hierarchy.*

I furthermore designed a forcing notion for stable Ramsey's theorem for pairs. This notion of forcing generalizes the first jump and second jump control techniques of Cholak, Jockusch and Slaman [8] and opens the door to the jump control of the Ramsey-type hierarchies.

A Ramsey-type weak König's lemma

After the proof by Liu [27] that $\mathsf{RCA}_0 \wedge \mathsf{RT}^2_2 \nvdash \mathsf{WKL}$, Flood [15] introduced a Ramsey-type weak König's lemma to clarify the relation between Ramsey's theorem for pairs and weak König's lemma. Informally, RWKL asserts the existence, for every non-empty Π^0_1 class $\mathcal{C}$, of an infinite set $H \subseteq P$ or $H \subseteq \overline{P}$ for some $P \in \mathcal{C}$. One has to state RWKL carefully not to imply the existence of a member of $\mathcal{C}$.

Montálban [31] introduced the informal notion of *robustness* of a statement. A theorem P is robust if its strength remains the same when considering slight variations of the statement. Robustness is an argument for the naturality of a statement. In particular, the Big Five are robust, and in some sense, so is weak weak König's lemma. Bienvenu, Shafer and I [3] studied extensively RWKL and showed that it exhibits robustness. In the following theorem, RCOLOR_k and RSAT are Ramsey-type versions of the infinite graph coloring problem and the infinite boolean satisfaction problem, respectively.

Theorem 12 ([3]). *For every* $k \geq 3$, $\mathsf{RCA}_0 \vdash \mathsf{RWKL} \leftrightarrow \mathsf{RCOLOR}_k \leftrightarrow \mathsf{RSAT}$.

Flood [15] proved that RWKL is a strict consequence of both SRT^2_2 and WKL. He furthermore proved that RWKL implies the diagonally non-computable principle, which asserts for every set X the existence of a function f such that $f(e) \neq \Phi^X_e(e)$ for every e. He asked whether DNC is a strict consequence of RWKL over RCA_0. Flood and Towsner [16] and Bienvenu, Shafer and I [3] independently clarified the relation between DNC and RWKL by proving the following theorems. By RWWKL, we mean the restriction of RWKL to trees of positive measure.

Theorem 13 ([3]). $\mathsf{RCA}_0 \vdash \mathsf{RWWKL} \leftrightarrow \mathsf{DNC}$.

Theorem 14 ([3]). $\mathsf{RCA}_0 \wedge \mathsf{WWKL} \nvdash \mathsf{RWKL}$.

Ramsey's theorem and finitistic reductionism

During the foundational crisis of mathematics, Hilbert proposed a program to justify the use of infinity, namely, *finitistic reductionism.* His goal was to reduce any proof of finite facts using infinitary method to a proof using only finitary ones. Although Gödel showed through his incompleteness theorems that such a program couldn't be realized in its full generality, Simpson [46] recently proposed a partial realization of Hilbert's program using the insights of reverse mathematics. Based on the work of Hilbert and Bernays [21] and of Tait [48], he formally interpreted finitistic reductionism as Π^0_1 conservation of subsystems of second-order arithmetic over primitive recursive arithmetics. Weak König's lemma being Π^0_1 conservative over PRA, a large part of mathematics can already proven to be reducible finitistically thanks to the program of reverse mathematics. The question whether Ramsey's theorem for pairs is finitistically reducible was a long-standing problem, until Yokoyama and I recently solved it.

Theorem 15 ([44])**.** RT^2_2 *is* Π^0_2 *conservative over* PRA*.*

A popularization of the result is presented in an article of Natalie Wolchover in Quanta Magazine [51].

Future work

Many open questions remain in the reverse mathematics of combinatorial theorems. My short term investigation will consist of trying to adapt the forcing notions of [42] to prove the strictness of the free set, thin set and the rainbow Ramsey theorem hierarchies. In particular, I will try to prove the following conjecture.

Conjecture 1. *Every* Δ^0_n *set has an infinite* low_n *subset in either it or its complement.*

I also would like to explore Hindman's theorem [1] further, which asserts for every coloring of the integers the existence of an infinite set over which the finite sums are monochromatic. Blass, Hirst and Simpson [4] proved that Hindman's theorem (HT) lies between ACA^+ and ACA. Montálban asked whether HT is provable in $\mathsf{RCA}_0 + \mathsf{ACA}$.

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